FX Cadzow / SSA Random Noise Filter: Frequency Extension

Summary
The application of FX Singular Spectrum Analysis (SSA) or Cadzow filtering approach for random noise attenuation on seismic data started from the works of Ulrych et al (1988) on eigenimage filtering of seismic data. Trickett furthered this work by applying eigenimage filtering to 3D data frequency slices and later extended FX Cadzow filtering by forming a larger Hankel matrix of Hankel matrices in multiple spatial dimensions (Trickett, 2003, 2009). We propose to add another dimension for creating extended Hankel matrices. Instead of using a single frequency slice for composing the extended matrix, we propose to use a range of frequency slices. This additional dimension of the matrix increases our statistics which improves the filter quality. The synthetic examples illustrate the filter quality improvement compared to the conventional FX Cadzow filter. Application of Frequency Extension (FE) filter in combination with FX Cadzow filter on real data showed better noise reduction compared to only FX Cadzow filtering.

Introduction
FX Singular Spectrum Analysis (SSA) has been successfully applied to seismic data. For application to seismic data, Trickett applied SSA separately on individual frequency slices. Later, following applications outside seismic processing - Golyandina (Golyandina et al, 2001, 2007) and Dologlou (Dologlou et. al, 1996), Trickett furthered this method by extending this FX Cadzow filtering (Cadzow, 1988) by means of forming a larger Hankel matrix of Hankel matrices in multiple spatial dimensions.

The purpose of this presentation is to introduce an additional dimension for composing the Hankel matrices. Instead of using only spatial dimensions for composing these matrices we propose to add a frequency dimension to create an extended matrix from a series of frequency slices. This Frequency Extension approach improves filter quality through better statistics by utilizing these multi-frequency slices.

Theory
The following two examples show how an extended block matrix A may be created:

a. Extended matrix

$$A = \begin{bmatrix} A_1 & A_2 & \cdots & A_n \\ A_2 & A_3 & \cdots & A_{n+1} \\ \vdots & \vdots & \ddots & \vdots \\ A_n & A_{n+1} & \cdots & A_{2n-1} \end{bmatrix} \qquad \text{where } A_i = \begin{bmatrix} a_{i,1} & a_{i,2} & \cdots & a_{i,n} \\ a_{i,2} & a_{i,3} & \cdots & a_{i,n+1} \\ \vdots & \vdots & \ddots & \vdots \\ a_{i,n} & a_{i,n+1} & \cdots & a_{i,2n-1} \end{bmatrix}$$

b. Extended matrix - an example

$$\begin{matrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{13} & a_{14} \\ a_{13} & a_{14} & a_{15} \end{bmatrix} & \begin{bmatrix} a_{21} & a_{22} & a_{23} \\ a_{22} & a_{23} & a_{24} \\ a_{23} & a_{24} & a_{25} \end{bmatrix} \\ \begin{bmatrix} a_{21} & a_{22} & a_{23} \\ a_{22} & a_{23} & a_{24} \\ a_{23} & a_{24} & a_{25} \end{bmatrix} & \begin{bmatrix} a_{31} & a_{32} & a_{33} \\ a_{32} & a_{33} & a_{34} \\ a_{33} & a_{34} & a_{35} \end{bmatrix} \end{matrix}$$

In Hybrid (C^2-FX) filtering (Trickett, 2009) or 2D-Extension (Golyandina et.al, 2007) a block matrix A is composed of sub-matrices (A_i) which may be constructed from neighboring shot gathers. Here each $a_{i,j}$ is a complex Fourier coefficient for trace *j* from ensemble *i*. After matrix construction, rank reduction is performed using singular value decomposition (SVD). Then Fourier coefficient $a_{i,j}$ for each trace is recovered from the rank-reduced matrix by averaging over multiple entries of the same $a_{i,j}$ in A, and an inverse Fourier transform returns the filtered traces.

Instead of composing these extended matrices only over space dimensions and for a single frequency slice, we propose to create extended matrices from several frequency slices. To illustrate this FE approach, let us use the same templates of the extended matrices as in examples above, but the meaning of subscripts will be different. In the example of the extended matrix (a), the second subscript *j* in $a_{i,j}$ will denote the same trace number as before, but the first subscript *i* will refer now to the sequential frequency slice number.

To show the validity of this FE filtering concept, let us consider a simple example in 2D FX domain. Given a two dimensional event with constant dip, it will be represented in TX and FX domains as follows:

$$u(t,x) = A\delta(t-px) \leftrightarrow U(\omega, x) = Ae^{-i\omega px} \qquad (1)$$

where *A* is an arbitrary coefficient representing the amplitude of the event, *t* is time, *x* is a spatial coordinate, ω is angular frequency and *p* is some coefficient representing the dip of the event. Let us assume that this data is regularly sampled $x_k = k\Delta x$ and $\omega_k = k\Delta\omega$. Sacchi (Sacchi, 2009) presented an example showing that the trajectory matrix built along the *x* coordinate in FX domain will have a rank of one, and, therefore, may be approximated by a rank one matrix in rank reduction.

Similarly, we can build a trajectory matrix along the ω coordinate in FX domain and such trajectory matrix will also have a rank of one.

Let the frequency series be

$$F_k = U(\omega_k, x) = Ae^{-i\,k\Delta\omega\,px} \qquad (2)$$

For 7 samples, the trajectory matrix M is

$$M = \begin{bmatrix} F_1 & F_2 & F_3 & F_4 \\ F_2 & F_3 & F_4 & F_5 \\ F_3 & F_4 & F_5 & F_6 \\ F_4 & F_5 & F_6 & F_7 \end{bmatrix} = \begin{bmatrix} Ae^{-i\,\Delta\omega\,px} & Ae^{-2i\,\Delta\omega\,px} & Ae^{-3i\,\Delta\omega\,px} & Ae^{-4i\,\Delta\omega\,px} \\ Ae^{-2i\,\Delta\omega\,px} & Ae^{-3i\,\Delta\omega\,px} & Ae^{-4i\,\Delta\omega\,px} & Ae^{-5i\,\Delta\omega\,px} \\ Ae^{-3i\,\Delta\omega\,px} & Ae^{-4i\,\Delta\omega\,px} & Ae^{-5i\,\Delta\omega\,px} & Ae^{-6i\,\Delta\omega\,px} \\ Ae^{-4i\,\Delta\omega\,px} & Ae^{-5i\,\Delta\omega\,px} & Ae^{-6i\,\Delta\omega\,px} & Ae^{-7i\,\Delta\omega\,px} \end{bmatrix} =$$

$$\begin{bmatrix} Ae^{-i\,\Delta\omega\,px} & Ae^{-2i\,\Delta\omega\,px} & Ae^{-3i\,\Delta\omega\,px} & Ae^{-4i\,\Delta\omega\,px} \\ e^{-i\,\Delta\omega\,px}\cdot Ae^{-i\,\Delta\omega\,px} & e^{-i\,\Delta\omega\,px}\cdot Ae^{-2i\,\Delta\omega\,px} & e^{-i\,\Delta\omega\,px}\cdot Ae^{-3i\,\Delta\omega\,px} & e^{-i\,\Delta\omega\,px}\cdot Ae^{-4i\,\Delta\omega\,px} \\ e^{-2i\,\Delta\omega\,px}\cdot Ae^{-i\,\Delta\omega\,px} & e^{-2i\,\Delta\omega\,px}\cdot Ae^{-2i\,\Delta\omega\,px} & e^{-2i\,\Delta\omega\,px}\cdot Ae^{-3i\,\Delta\omega\,px} & e^{-2i\,\Delta\omega\,px}\cdot Ae^{-4i\,\Delta\omega\,px} \\ e^{-3i\,\Delta\omega\,px}\cdot Ae^{-i\,\Delta\omega\,px} & e^{-3i\,\Delta\omega\,px}\cdot Ae^{-2i\,\Delta\omega\,px} & e^{-3i\,\Delta\omega\,px}\cdot Ae^{-3i\,\Delta\omega\,px} & e^{-3i\,\Delta\omega\,px}\cdot Ae^{-4i\,\Delta\omega\,px} \end{bmatrix}$$

this illustrates that the frequency extension matrix M has a rank of one.

In the example above, assuming that we have only one linear event and random noise, FE filtering means

(1) find matrix M_1 of rank $r=1$ nearest, in a least square sense, to the original trajectory matrix *M* composed of a range of Fourier slices of the observed data

(2) recover FE filtered traces by taking the inverse Fourier transform of the Fourier coefficients in M_1.

M_1 will have multiple entries of Fourier coefficients corresponding to the same trace and an average value of them will be used for the inverse Fourier transform in the same manner as in C^2-FX filtering.

Similarly, it is possible to create the FE filter with higher ranks and for extended block matrices.

Synthetic Examples
The synthetic examples (Fig. 1-5) show the comparisons between the standard Cadzow FX filter and FE filtering approach. For FE filtering we used 2D extension in both spatial and frequency dimensions. The frequency extension filter result looks cleaner than that after the Cadzow FX filter and its difference display shows that more noise has been removed.

Figure 1: Input data

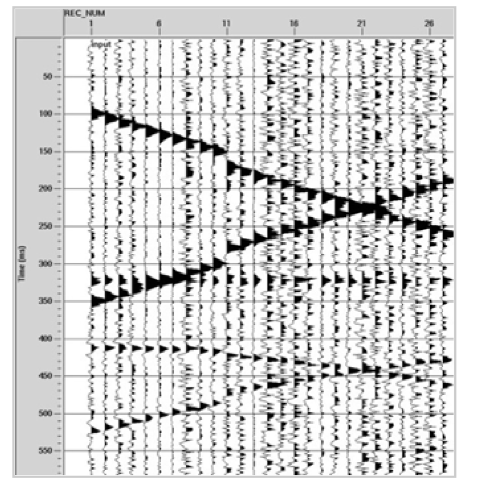


Figure 2: FX Cadzow filter, rank 2

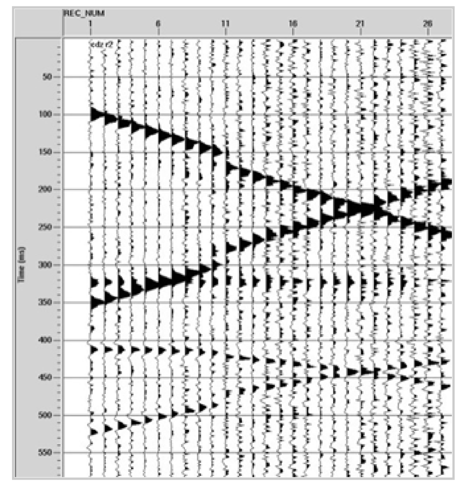


Figure 3: FE filter, rank 3

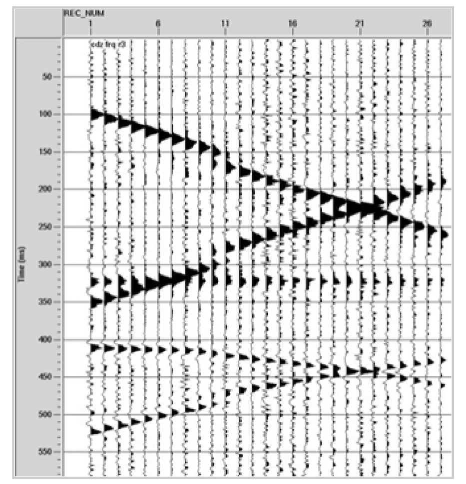


Figure 4: Difference plot:
Input – FX Cadzow

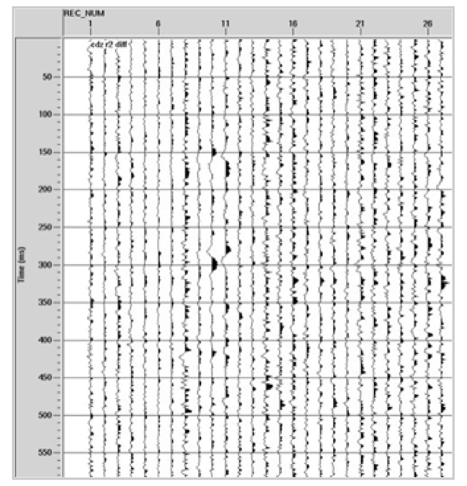
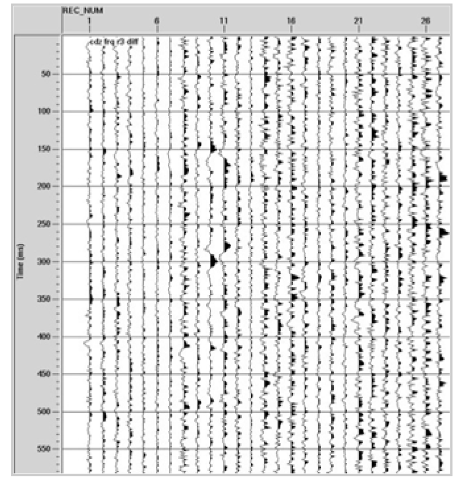
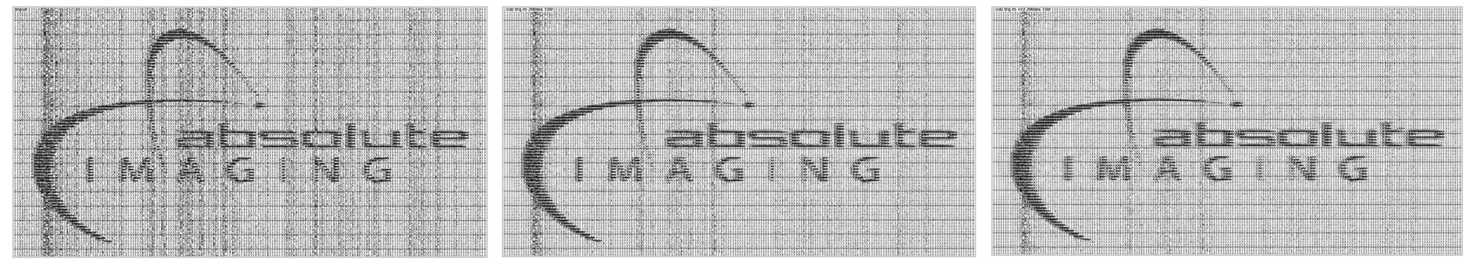


Figure 5: Difference plot:
Input – FE filter



The following synthetic examples in Figure 6 show that the FE filter produces better results when applied together with the Cadzow FX filter. The FE filter was used as a pre filter in order to improve the final output. In this case we applied a mild FE filter (rank 5) followed by a harsher Cadzow FX (rank 3). Figure 6 shows that the result of such a combined application of the two filters FE + Cadzow FX is more effective than Cadzow FX filtering alone.

Figure 6: More synthetic examples



Input data | Cadzow FX filter | Filter series: FE - Cadzow FX

Real Data Examples
The Frequency Extension approach proposed in this paper was applied to 2D shot ensembles. Figure 7 shows one of the original shot records ordered by offsets, the results of application on this record C^2-FX filter (ranks 2 and 3), FE filter (rank 4), FE combined with C^2-FX filter, and the difference plots. On these data, the results of Frequency Extension filtering were comparable to that for C^2-FX filter.

However, when both filters are applied together, the shot record looks better – FE rank 4 followed by C^2-FX rank 2. Like in synthetic data example, the basis for selection of parameters for these filters (ranks) was to apply a mild filter first (FE with rank 4), and then apply a stronger second filter (C^2-FX with rank 2).

The results of stacking these data are shown in Figures 8-10: unfiltered structure stack, stack after application of C^2-FX filtering on shots and stack after application of FE combined with C^2–FX on shots. The application of C^2-FX filtering (Fig. 9) look much better then unfiltered stack (Fig. 8), as it was expected, but combined filtering with FE and C^2-FX (Fig. 10) is cleaner than stack after only C^2-FX filtering.

Figure 7: Ensemble and Frequency Extension Cadzow filtering and difference plots

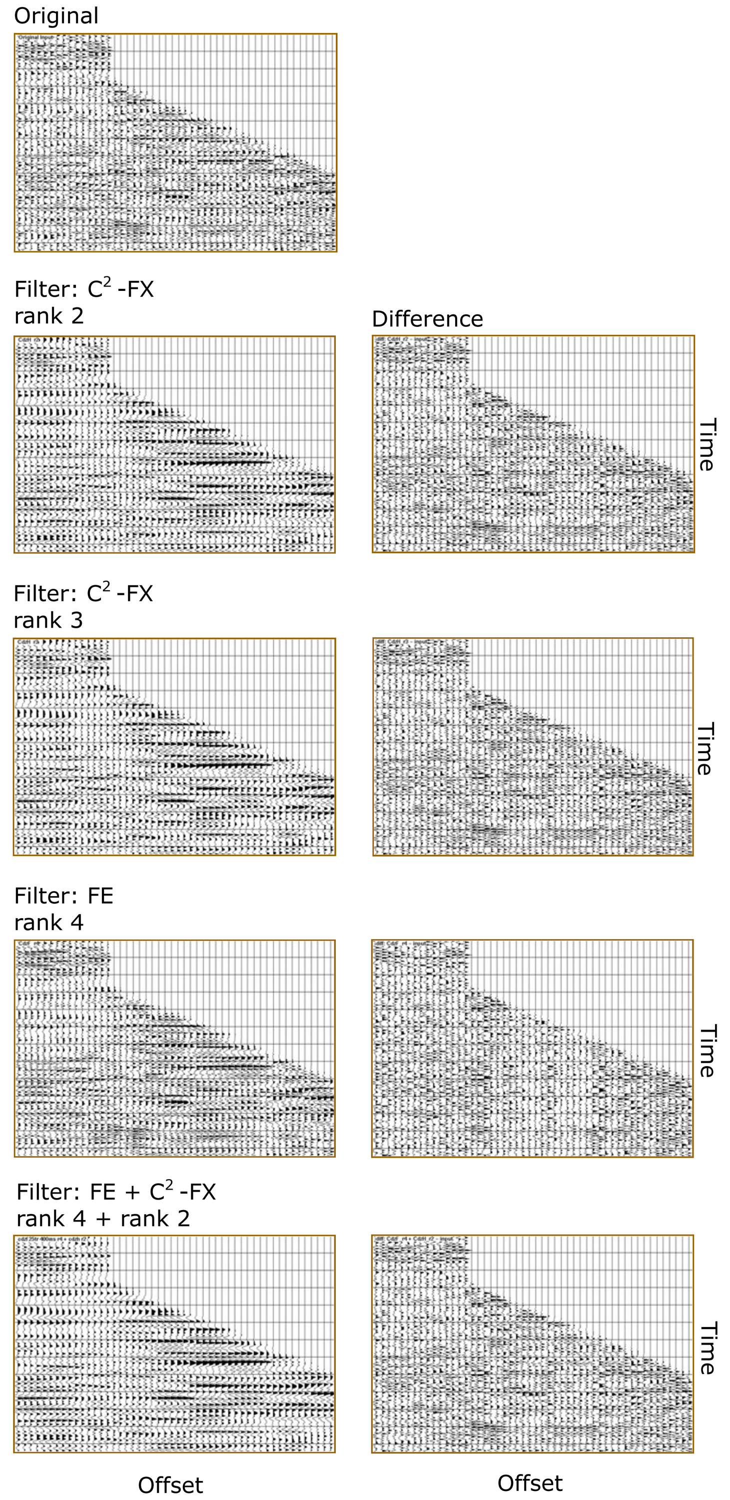


Figure 8: Structure stack

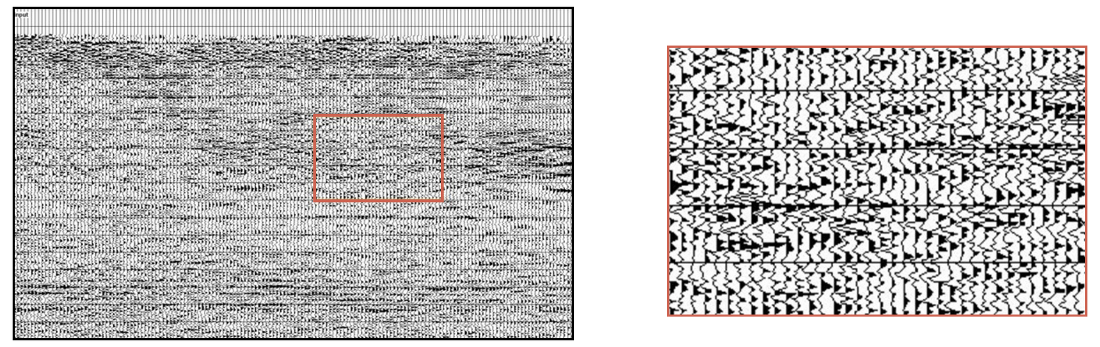


Figure 9: Structure stack: C2-FX (rank 2) on shots

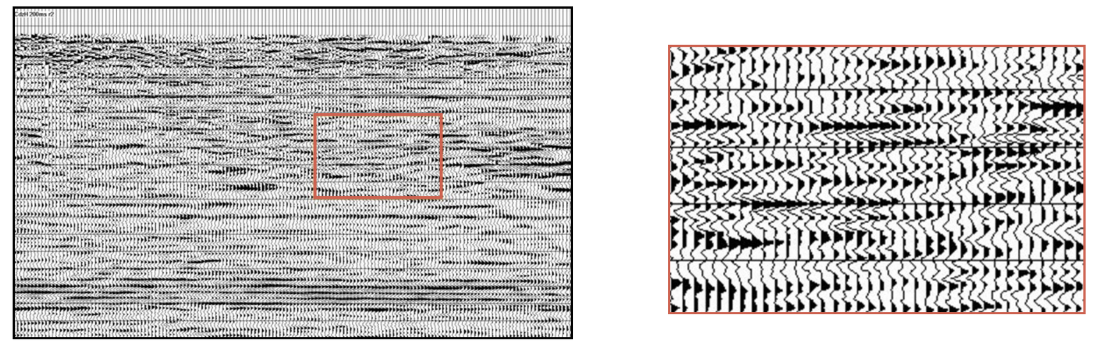
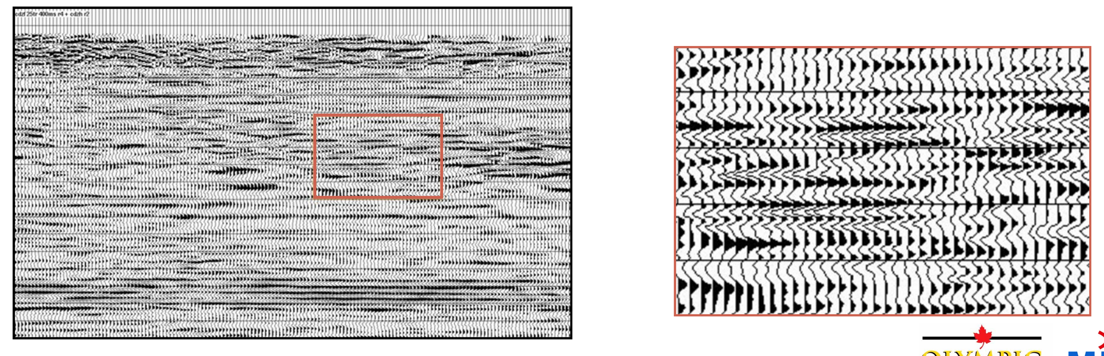


Figure 10: Structure stack: FE (rank 4) combined with C2–FX (rank 2) on shots



Seismic data courtesy of: OLYMPIC SEISMIC, MURPHY OIL COMPANY LTD.

Conclusion
The results of using an additional frequency dimension in composing the extended matrices of the Cadzow FX filters look promising. Application of proposed Frequency Extension filter together with C^2 –FX filter on the real 2-D data showed cleaner shot records and stack compared to only C^2 –FX filtering. As in the Hybrid Cadzow, this FE approach opens the way for composing larger extended matrices by utilizing more spatial coordinates which should result in a more accurate filter.

References:

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