Summary
The application of the Singular Spectrum Analysis (SSA) or CSD Cadzow filtering approach for random noise reduction has been thoroughly examined. An alternative way to extract matrices from frequency slices was introduced in this paper. This approach enables to form a larger Hankel matrix of Hankel matrices in multiple spatial dimensions (Trickett, 2003, 2009). The extended Hankel matrices are applied together with the Cadzow FX filter. This combination improves the filter quality. The synthetic examples illustrate the filter quality improvement compared to the conventional FX Cadzow filter.

Introduction
The Singular Spectrum Analysis (SSA) has been successfully applied to seismic data. For application to both spatial and frequency domains, SSA was proposed by Canales (1984). SSA was extended to a time-frequency domain by utilizing multiple linear events in a least square sense. The idea of forming a larger Hankel matrix of Hankel matrices in multiple spatial dimensions was proposed by Gaubitch and Papanicolaou (2001) and Gaubitch et al (2007). This approach enables to form a larger Hankel matrix of Hankel matrices in multiple spatial dimensions.

The purpose of this presentation is to introduce an additional dimension for composing the Hankel matrices. Instead of using only spatial dimensions for composing these matrices, we propose to add a frequency dimension to create an extended matrix from a series of frequency slices. This Frequency Extension (FE) filter in combination with the CSD Cadzow filter on real data showed better noise reduction compared to only the Cadzow filtering.

Theory
The following two examples show how an extended block matrix A may be created.

A. extended matrix

\[
\begin{bmatrix}
A_{11} & A_{12} & \cdots & A_{1r} \\
A_{21} & A_{22} & \cdots & A_{2r} \\
\vdots & \vdots & \ddots & \vdots \\
A_{r1} & A_{r2} & \cdots & A_{rr}
\end{bmatrix}
\]

where \( A_{ij} \) is a Hankel matrix

B. extended matrix - an example

\[
\begin{bmatrix}
A_{11} & A_{12} & A_{13} & A_{14} \\
A_{21} & A_{22} & A_{23} & A_{24} \\
A_{31} & A_{32} & A_{33} & A_{34} \\
A_{41} & A_{42} & A_{43} & A_{44}
\end{bmatrix}
\]

In hybrid (FX) filtering (Trickett, 2009) or 3D filtering (Gaubitch et al., 2007) a black matrix A is created by matrices (A_i) which may be constructed from filtering data (data at each i) and, therefore, may be approximated by a rank one matrix in rank reduction.

Syntethic Examples
The synthetic example (Fig. 2) shows the comparison between the standard CSD Cadzow filter and FE filtering approach. For FE filtering we add 2D extension in both spatial and frequency dimensions. The frequency extension matrix codec gives better results than the FE filter and the difference displays below that main noise has been removed.

Summary
We propose to add another dimension for creating extended Hankel matrices. This Frequency Extension approach improves the filter quality. The synthetic examples illustrate the filter quality improvement compared to the conventional FX Cadzow filter. Application of Frequency Extension filter in combination with the CSD Cadzow filter on real data showed better noise reduction compared to only the Cadzow filtering.

References
Canales, L.L., 1984, Random Noise Reduction: SEG Extended Abstracts, 525