A Practical One Way Scalar Wavefield Extrapolation: A Step Forward in True Amplitude Processing

Summary
A new Phase-shift method for wavefield downward continuation is presented. Unlike conventional Phase-shift which is based on the one way scalar wave equation that is factorized from homogeneous media, this method is derived by directly solving the full scalar wave equation to obtain one way wave propagation in an inhomogeneous media.

Introduction
Propagation of wavefield in an inhomogeneous media causes reflection and transmission energy. A two layer example is shown in figure 1a. Methods employed for wave equation migration such as Phase-shift (Gazdag, 1978), are based on wavefield propagation in homogeneous media, therefore true amplitude during wavefield extrapolation cannot be obtained as shown in figure 1b. In this paper we present a method for true amplitude extrapolation as shown in figure 1c.

Methods
The scalar wavefield propagation problem (constant density) is defined with the wave equation (2D case)

\[ \frac{\partial^2 u}{\partial t^2} - \nabla \cdot (\mu \nabla u) = \rho \ddot{u} (x, y, z, t) = 0 \]

where:

- \( u(x,y,z,t) \) = wavefield
- \( x, y \) = spatial coordinates
- \( \ddot{u} \) = acceleration
- \( \mu \) and \( \rho \) = material parameters

Displacement and stress representation of equation above (Kosloff and Byasal, 1983, 1987): is:

\[ \frac{\partial^2 \Phi}{\partial t^2} - \nabla \cdot (\mu \nabla \Phi) + \rho \Phi = 0 \]

where:

- \( \Phi \) = wavefield
- \( \mu \) and \( \rho \) = material parameters

Eigen value decomposition of matrix \( H \):

\[ H = \Gamma \Lambda \Gamma^{-1} \]

where:

\[ F = \left[ \begin{array}{c} F_0 \\ \vdots \\ F_N \end{array} \right] = \Gamma^{-1} \left[ \begin{array}{c} F_0' \\ \vdots \\ F_N' \end{array} \right] \]

by defining:

\[ F' = \phi(x,z) \]

we have:

\[ \frac{\partial F'}{\partial t} = \Gamma^{-1} \frac{\partial F'}{\partial t} \]

and:

\[ R = F'(x,z) / F'(x,z) \]

Until now no approximation has been made. However, the coupled terms still make the practice difficult. By ignoring multi-scattering we have:

\[ R = \frac{1}{2} S_0 + R_E \]

It is of interest to get an insight on the reflection term \( R \) in the equation above. If \( F' \) can be replaced by \(-i\) via a Fourier Transform as in the case of a laterally homogeneous media then:

\[ R = \frac{1}{2} \frac{1}{2} S_0 + R_E \]

and it is equivalent to the WKBJ approximation result (Zhang, 2005). Furthermore, in the case of a layered media, the interval between layers \( k \) and \( k+1 \) can be defined as:

\[ R = \frac{1}{2} S_0 + R_E \]

which is the reflection coefficient formula for an acoustic wave (Ceverny, 2001).

Practical Implementation for Migration
With the assumption that the vertical velocity does not change within each extrapolation step then:

\[ \frac{\partial L}{\partial t} + i \omega \mu L + R_E \Phi (z-z') \]

With this equation, the reflected energy for the backward wavefield does not participate in further forward propagation but will be included in the wavefield for further backward propagation.

Synthetic Example
Figure 2 shows a shot gather with a simple velocity model shown in figure 1. Receiver stations are 30 meters apart and the two reflectors are located at depths of 1000 and 2000 meters respectively.

Discussion
A new formula for one way wave equation extrapolation is presented. Comparing this formula with the conventional Phase-shift method shows that they are very similar except for an extra term related to the reflection. Numerical results for a 2 layer model shows that applying this method to migration can produce a more correct image coincident with the reflection coefficient.

References:

- Kosloff, R., and Kessler, D., 1983 : Migration with the full acoustic wave equation, Geophysics V.48, 667
- Gazdag, J., 1978 : Migration of seismic data by phase shift plus interpolation, Geophysics, V.49, 124
- Ceverny, J., 2001 : Theory of true amplitude wavefield extrapolation: A step forward in true amplitude processing, Geophysics, V.66, 893
- Wenzel, R., 1991 : Frequency domain split-step Fourier migration, Geophysics, V.56, 1547
- Zhang, Y., Zhang, G., and Bleistein, N., 1993 : Theory of true amplitude wavefield extrapolation and true amplitude migration, Geophysical Prospecting, V.41, 667
- Kosloff, R., and Kessler, D., 1983 : Migration with the full acoustic wave equation, Geophysics V.48, 667
- Gazdag, J., 1978 : Migration of seismic data by phase shift plus interpolation, Geophysics, V.49, 124
- Ceverny, J., 2001 : Theory of true amplitude wavefield extrapolation: A step forward in true amplitude processing, Geophysics, V.66, 893

www.absoluteimaging.ca

Presented by: Zhengsheng Yao
www.absoluteimaging.ca